43.2: Entropy-Constrained Error Diffusion

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ABSTRACT

While traditional error diffusion generates very high quality halftones from continuous tone images, the resulting binary images are not amenable to lossless compression. We propose an algorithm that incorporates an entropy constraint and a delayed decision procedure, for generating halftone images that are more amenable to lossless compression. In essence, we tradeoff distortion with compression ratio as in rate-distortion theory.

1. INTRODUCTION

Error diffusion [1–6] is very popular for generating high quality halftones from continuous tone images. While error diffusion generally produces halftones of good quality, the resulting halftones are not amenable to lossless compression, *i.e.,* the compression ratio that we can achieve using lossless coders is generally very low. For a halftone of the Lena image halft oned by Floyd-Steinberg error diffusion, we can achieve a compression ratio of about 1.7 using a standard JBIG lossless coder [7].

If we view the halftoning procedure as a coding problem where the output alphabet size of the code is constrained to be two (black or white), then we can invoke rate distortion theory [8] in designing a halftoning algorithm that trades compression performance with distortion. For a tree coding halftoner [9], which generates a hi-level image by minimizing a cost function (typically distortion), one can introduce an entropy constraint to trade distortion with compression ratio [10]. Similar approach can be applied in any other optimization based halftoning algorithm. There is, however, no cost function associated with traditional error diffusion [1], and hence there is no obvious way to incorporate an entropy constraint.

In [11], the concept of delayed decision is introduced into the error diffusion algorithm, where one considers at each pixel location a sequence of future pixels conditioned on the value of the current possible binary output, and then decide on the binary output pixel value using a distortion criterion. In this paper, we incorporate an entropy constraint into the cost function, and extend the delayed decision error diffusion algorithm to obtain halftones that optimally trade off between compressibility and image quality.

2. DELAYED DECISION ERROR DIFFUSION

We first briefly review the delayed decision error diffusion algorithm that was developed in [11]. To describe the algorithm, we assume that we are currently at the pixel location (m, n), where all the "previous" output pixels with respect to the scanning strategy have been determined. To decide whether the current output pixel $b_{m,n}$ should be a "0" or a "1, " we first suppose that the output is "O, " and denote the output pixels under this condition by $b^{(0)}_{m,n}$. Hence we have

$$b^{(0)}_{k,l} = \begin{cases} b_{k,l} & \text{if } (k,l) < (m,n) \\ 0 & \text{if } (k,l) = (m,n), \end{cases}$$

where we say $(k,l)<(m,n)$ if the pixel location (k,l) is processed before (m,n) according to the scanning strategy. We then continue as usual with the traditional error diffusion process for L steps, and obtain the output $b^{(0)}_{m,n+j}$ for $j = 1,2,\ldots,L$. After these L steps, we calculate' the distortion

$$\delta^{(0)}_{m,n} = \sum_{j=0}^{L} \left(\left(x_{m,n+j} - (v * b^{(0)})_{m,n+j} \right)^2 + \gamma u^{(0)}_{m,n+j} \right)$$

where $*$ denotes convolution, $v_{k,l}$ is a causal impulse response that approximates the characteristics of the human visual system, $u^{(0)}_{m,n}$ is a distortion based on the spatial distribution of the pixels $b^{(0)}_{m,n}$, and γ is a weighting parameter. Specifically, we have

$$u^{(0)}_{m,n} = \begin{cases} 0 & \text{if } d^{(0)}_{m,n} \geq d_p(x_{m,n}) \\ & \quad \text{and } b^{(0)}_{m,n} = \rho_{m,n} \\ 0 & \text{if } d^{(0)}_{m,n} < d_p(x_{m,n}) \\ & \quad \text{and } b^{(0)}_{m,n} \neq \rho_{m,n} \\ \left(\dfrac{d_p(x_{m,n}) - d^{(0)}_{m,n}}{d_p(x_{m,n})} \right)^2 & \text{otherwise,} \end{cases} \tag{1}$$

where $\rho_{m,n}$ denotes the value of the minority pixels [4], $d_p(x_{m,n})$ is the principal dist ante [4] corresponding to $x_{m,n}$, and $d^{(0)}_{m,n}$ is the actual distance from location (m,n) to the nearest minority pixels in the bitmap $b^{(0)}_{m,n}$. Details of the

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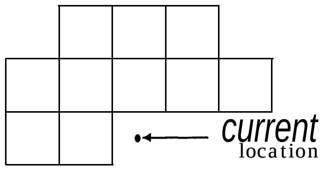

Figure 1. Template of previous output pixels used as a context for computing conditional entropy.

mixture distortion can be found in a recent paper [9]. Note that the calculation of $\delta^{(0)}_{m,n}$ requires both the past output pixel values, *i.e.*, $b_{k,l}$ for $(k,l) < (m,n)$, as well as the future L output pixel values conditioned on the output at (m,n) being equal to "0," *i.e.*, $b^{(0)}_{m,n+j}$ for $j = 1,2,\ldots,L$. Following this, we consider the other case where the output at (m,n) is "1," and denote the output sequence under this condition by $b^{(1)}_{m,n}$. Similarly we have

$$b^{(1)}_{k,l} = \begin{cases} b_{k,l} & \text{if } (k,l) < (m,n) \\ 1 & \text{if } (k,l) = (m,n), \end{cases}$$

We then generate the next L output values $b^{(1)}_{m,n+j}$ for $j = 1,2,\ldots,L$ using generic error diffusion, and calculate

$$\delta^{(1)}_{m,n} = \sum_{j=0}^{L} \left(\left(x_{m,n+j} - (v * b^{(1)})_{m,n+j} \right)^2 + \gamma u^{(1)}_{m,n+j} \right).$$

Using $\delta^{(0)}_{m,n}$ and $\delta^{(1)}_{m,n}$, we determine the actual output at location (m,n) by

$$b_{m,n} = \begin{cases} 0 & \text{if } \delta^{(0)}_{m,n} \le \delta^{(1)}_{m,n} \\ 1 & \text{otherwise.} \end{cases}$$

In other words, we choose the binary output value at location (m,n) that results in a smaller cumulative distortion L steps into the future. After making the decision, we discard the future bit values of $b^{(0)}_{m,n}$ and $b^{(1)}_{m,n}$, move on to the next pixel location, and repeat the same procedures to generate the next output sample. This is continued until the entire image has been processed.

3. ENTROPY CONSTRAINT

To use an entropy constraint for optimally trading distortion with compressibility in the output halftones, we minimize a cost function of the form

$$\tilde{J}_{m,n} = \delta_{m,n} + \lambda h_{m,n}(c_{m,n})$$

where $\delta_{m,n}$ is a mixture distortion measure as defined in the previous section, $h_{m,n}(\cdot)$ is an entropy measure, and $c_{m,n}$ is a *context* [12] defined by a window of neighboring pixels. In other words, $h_{m,n}$ is actually a conditional entropy measure conditioned on the value of the context. The quantity λ is a parameter that determines the location of the resulting halftone on the *operational rate-distortion* function.

To ensure good performance at a reasonable complexity, we compose the context using a template of 10 pixels as shown in Fig.1. As a result, there are 1024 different possible contexts. Since the output is binary, the conditional entropy is completely determined by

$$p_{m,n}(\beta,c) = \Pr\{b_{m,n} = \beta | c_{m,n} = c\} \qquad \beta = 0,1.$$

These probabilities are then estimated using the statistics of the past output pixels, and are continuously updated as the halftone image is being generated. Initially when no data is available, all the conditional probabilities are set to 0.5 so that neither "0" or "1" is favored. Using the estimates of the conditional probabilities, we can minimize

$$J_{m,n} = \delta_{m,n} - \lambda \log(p_{m,n}(b_{m,n}, c_{m,n})),$$

where it is known that the term $-\log(p_{m,n}(b_{m,n}, c_{m,n}))$ approximates the average length of a code word required to describe $b_{m,n}$.

The entropy constrained delayed decision error diffusion algorithm can be represented by the flow diagram in Fig.2. Instead of using the distortions $\delta^{(0)}$ and $\delta^{(1)}$ as a basis of making a decision, we use the cost functions

$$J^{(0)}_{m,n} = \delta^{(0)}_{m,n} - \lambda \log(p_{m,n}(0, c_{m,n}))$$

and

$$J^{(1)}_{m,n} = \delta^{(1)}_{m,n} - \lambda \log(p_{m,n}(1, c_{m,n})).$$

We determine the actual output at location (m,n) by

$$b_{m,n} = \begin{cases} 0 & \text{if } J^{(0)}_{m,n} \le J^{(1)}_{m,n} \\ 1 & \text{otherwise.} \end{cases}$$

Based on the choice for $b_{m,n}$, we update the statistics for the conditional probability. As in the delayed decision error diffusion algorithm, we discard the future bits of $b^{(0)}_{m,n}$ and $b^{(1)}_{m,n}$, move on to the next pixel location, and repeat the same procedures until the entire image has been processed.

4. EXPERIMENTAL RESULTS

We implemented the entropy constrained error diffusion algorithm on an HP 735 workstation. The coefficients of the filter $v_{m,n}$ are

		0.1680	0.1207	0.0549
0.0519	0.0928	0.1207	0.0928	0.0519
0.0438	0.0519	0.0549	0.0519	0.0438

Note that the (0,0) element is 0.1680. We found experimentally that setting $\gamma = 0.03$ gives the best halftone image quality. As mentioned in the previous section, the value of λ controls the compression performance (and also the image quality) of the generated halftone. The conditional probabilities are updated at each pixel location by

$$p_{m,n}(\beta,c) = \frac{N_{m,n}(\beta,c)+1}{N_{m,n}(c)+2} \qquad \beta = 0,1$$

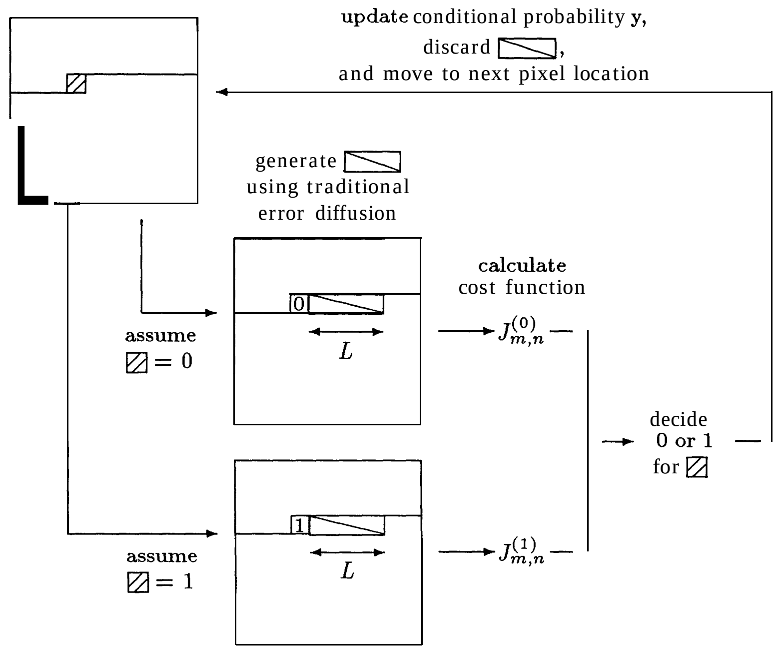

Figure 2. Flow diagram of the entropy constrained delayed decision error diffusion algorithm.

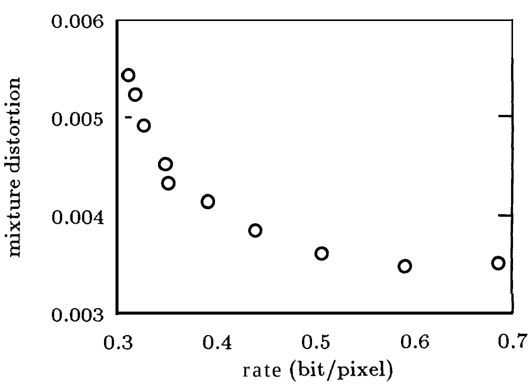

Figure 3. Rate and distortion performance of entropy constrained error diffusion with $L = 3$.

where $N_{m,n}(c)$ is the number of times the context c has occurred up to the pixel location (m, n), while $N_{m,n}(\beta, c)$ is the number of times that the context c is followed by the bit value β.

We have generated halftones using various values of λ, and plotted the compression versus distortion performance in Fig. 3 for $L = 3$. In this case, it takes about 17 seconds for our implementation to generate a halftone of size 512 by 512 pixels using an HP 735 workstation. Figures 4 through 6 shows the output of the delayed decision error diffusion algorithm at different levels of rate and distortion. The compression ratios achieved using a JBIG coder for Figures 4, 5 and 6 are 1.46, 2.28, and 3.06, respectively. As expected, one can adjust λ to obtain the desired the image quality or compression ratio.

Figure 4. Lena halftoned by entropy constrained error diffusion. Compression ratio achieved using JBIG is 1.46. The printing resolution is 150 dpi.

Figure 5. Lena halftoned by entropy constrained error diffusion. Compression ratio achieved using JBIG is 2.28. The printing resolution is 150 dpi.

Figure 6. Lena halftoned by entropy constrained error diffusion. Compression ratio achieved using JBIG is 3.06. The printing resolution is 150 dpi.

5. CONCLUSION

We have incorporated an entropy constraint into the delayed decision error diffusion algorithm to optimally trade off between image quality and compression performance. The resulting halftones are suitable for transmission over communications with a rate constraint. We have also presented experimental results indicating the quality levels that can be obtained at various rates.

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