43.1: A Study on Entropy Reduction of Bi-Level Color Image Quantized by the Minimized Average Error Method

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Abstract

The most efficient template of reference pixels in Markov-model encoding of a color image quantized by the hi-level Minimized Average Error Method (MAEM) is investigated, and the resultant Markov entropy is evaluated. Its efficacy in reducing the entropy is confirmed.

Introduction

Recently, image half-toning technique is widely studied, since it is indispensable for transmitting gray-scale images through facsimile communication. This technique is also important for its applicability to color images, because most color printing devices now widely spread have only two-tone (ON & OFF) printing capability in each color component. Among the quantizing techniques, one of the most popular methods is the Minimized Average Error Method (MAEM), because it has excellent performance in rendering both photographic images and texts[1].

Efficient encoding of a half-tone image quantized by the MAEM is therefore an important issue for short time transmission and/or saving storage capacity. Regarding an image as an output of binary Markov source is one of the very efficient encoding scheme, and it is important to find the context which gives less entropy under the given number of Markov-states.

In this paper, we discuss a technique which provides an efficient encoding scheme for half-tone images quantized by the MAEM. First, we examine a method to obtain a quasi-optimal template of reference pixels in monochrome image, and evaluate its performance in reducing the entropy. Second, we apply this method to obtaining a common template of reference pixels which is applicable to a wide variety of monochrome images. Finally, we apply the method to obtaining the template for hi-level color images by exploiting the correlation across multiple color components, and introduce the most efficient encoding sequence for minimizing the total transmission time of three color components.

The method to obtain the quasi-optimal template

In the MAEM, the original gray-level of the pixel at (x,y), denoted as $I_{x,y}$, is first compensated with the rendition errors in the neighboring pixels, and then quantized with a fixed value of 1/2. The compensation is done by adding the weighted sum of rendition errors $E_{x-i,y-j}$ incurred by thresholding preceding pixels to the pixel at (x,y). The compensated level, denoted as $I'_{x,y}$ is therefore calculated through the following equations:

$$I'_{x,y} = I_{x,y} + (1/\Sigma\, \alpha_{i,j})\, \Sigma\, \alpha_{i,j} E_{x-i,y-j}\,,$$
$$E_{x,y} = I'_{x,y} - P_{x,y}\,,$$

where $\alpha_{i,j}$ denotes the weighting coefficients, e.g. Jarvis' [2], and $P_{x,y}$ denotes the thresholded value (0 or 1) of $I'_{x,y}$.

Generally, the conditional entropy Hc is defined by

$$Hc = -\Sigma P(Si)\, \Sigma P(Xj/Si) \log P(Xj/Si),$$

where P(Si) is the probability of the occurrence of the state Si defined by the combination of the values of reference pixels Xj-k's (k=1,... i) in the template.

Now, we define a "base window" which consists of 17 candidate pixels for a template of reference pixels (See Figure 1). Out of the base window, a set of N reference pixels, which we call "N-reference template" hereafter, will be selected as a suitable template of N reference pixels. The N-reference template may be any combination of N pixels out of 17 pixels in the base window.

R00	R01	R02	R03	R04	R05	R06
RIO	R11	R12	R13	R14	R15	R16
R20	R21	R22	Xj			

Figure 1. Base window

In our study, however, to reduce the amount of calculation load, we introduce an assumption that the best N-reference template includes all of N-1 pixels selected as the best (N-1)-reference template. With this assumption, we can obtain the N-reference template of any N in a recursive manner for each image with a small calculation load[3].

For the recursive calculation, two methods can be considered: one is increasing the number of pixels in the reference template from 1 to 17, and the other is decreasing the number from 17 to 1. We call hereafter these two methods "Extending Pixel Method (EPM)" and "Reducing Pixel Method(RPM)", respectively.

For the simulation, we used several images from JIS-SCID (Japanese Industrial Standards - Standard Color Image Data) images (2048dots X 2560 dots). A luminance component is synthesized from three color components and half-toned with the MAEM. Figure 2 and 3 show the resultant priority order of the reference pixels in case of the image "cafeteria" obtained from these two methods. In these figures, the number in each box means the priority level of the relevant reference pixel. For example, the set of pixels located in the boxes labeled "01", "02" and "03" is the best combination of three reference pixels in the base window. Figure 4 shows the calculated conditional entropies of hi-level images synthesized from three SCID images ("bicycle", "cafeteria" and "portrait"), conditioned by the N-reference templates of the EPM and the RPM. From this result, it is observed that the EPM shows better performance than the RPM.

We also calculate the conditional entropies for every possible combination of N reference pixels in the base window, and find the best template of reference pixels. We call this method "Full Search Method (FSM) ". Table 1 shows the entropies of each image conditioned by the two 12-reference templates obtained from the FSM and

14	10	9	12	13	15	16
3	5	2	4	7	6	17
1	11	8	Xj			

Figure 2. N-reference template of EPM

12	9	6	12	11	5	8
2	10	3	4	13	17	7
1	15	16	Xj			

Figure 3. N-reference template of RPM

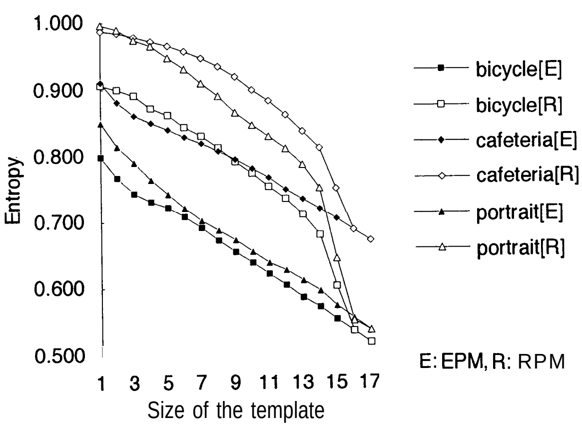

Figure 4. Conditional entropies with N-reference templates of EPM and RPM

Table 1. Conditional entropies with the 12-reference templates of FSM and EPM [bits/pel]

	FSM	EPM	**FSM/EPM**
bicycle	0.602	0.607	0.992
cafeteria	0.748	0.748	1.000
fruits	0.634	0.645	0.983
orchid	0.545	0.586	0.930
portrait	0.610	0.627	0.973
wine	0.591	0.608	0.972
average	0.622	0.637	0.975

the EPM. From this table, it can be observed that the increase of entropy by using the EPM instead of the FSM, is less than 3% except the case of "orchid", and even that in case of "cafeteria" the reference template obtained from the EPM is identical to that from the FSM. It is concluded from this observation that the EPM can be regarded as a sufficient alternate of the FSM, and that we can use the EPM to obtain the reference templates of any number of reference pixels.

The common template of reference pixels

Next, we investigate the applicability of the EPM to obtaining a reference template common to a variety of images. We synthesize a concatenated image of 6 SCID images ("bicycle", "cafeteria", "fruits basket", "orchid", "portrait" and "wine"), and obtain "common N-reference templates" by the EPM. Figure 5 shows the order of pixels, like Figure 2, which identifies the common N-reference templates for N=1 to 17. In Table 2, the entropies conditioned by the common 12-reference template are compared with those conditioned by the 12-reference templates optimized by the FSM for each image. From this table, it is observed that the increase of entropy is only up to 3%, except the case of "orchid", and therefore, it is concluded that the common reference templates obtained by the EPM is applicable to a wide range of hi-level images quantized by the MAEM.

17	16	15	8	12	6	7
2	3	4	9	13	14	10
1	11	5	Xj			

Figure 5. Common N-reference template

Table 2. Conditional entropies with the 12-reference template obtained by FSM and the common 12-reference template by EPM

	FSM	common	**FSM/common**
bicycle	0.602	0.613	0.982
cafeteria	0.748	0.764	0.979
fruits	0.634	0.648	0.978
orchid	0.545	0.581	0.938
portrait	0.610	0.619	0.985
wine	0.591	0.601	0.983
average	0.622	0.638	0.974

Application to color images

Component specific templates for hi-level color images

We also apply the EPM to obtaining a quasi-optimal template which is applicable to each component of hi-level color images quantized by the MAEM. We choose Blue(B), Green(G) and Red(R) color components for their wide applicability. In each color component, three SCID images ("bicycle", "cafeteria" and "portrait") are concatenated, and an N-reference template for each component is obtained as the "component specific N-reference template" (template S). Table 3 shows the entropy of each color component of the image "cafeteria" conditioned by the component specific N-reference template. Like in case of luminance component, it is confirmed that the increase of entropy by applying the EPM instead of the FSM is up to a few percents. From this result, it can be said that the EPM is also applicable to obtaining a component specific N-reference template for each component of three hi-level color images.

Table 3. Conditional entropies with the component specific 12-reference templates [bits/pel]

cafeteria	S(FSM)	S(EPM)	FSM/EPM
B	0.701	0.710	0.988
G	0.792	0.798	0.992
R	0.749	0.752	0.996
average	0.747	0.753	0.992

EPM with multiple color components

Generally, color components of an image are strongly correlated with each other. Assuming that it is true even in case of hi-level images half-toned by MAEM, we examine an N-reference template allowing the pixels in the different color component as the candidates of reference pixels. Now we define another "base window" which consists of 15 candidate pixels in a color component other than the one currently encoded (See Figure 6). In this new "base window", the box labeled "D12" corresponds to the pixel located at the same position of the pixel "Xj" in the color component currently encoded. In the EPM with multiple color components, N-reference pixels are selected from the candidate pixels included in these two base windows. We call the N-reference template obtained from the EPM with multiple color components a "multi-component N-reference template", hereafter.

Now we assume that three color components of an image are transmitted in the order of B, G and R by plane interleaving. The multi-component N-reference template for G or R component can therefore include reference pixels in B or G component, respectively, which is transmitted earlier. On the other hand, the N-reference template for B component can only include previously transmitted pixels within the component. Figure 7 shows the resultant multi-component 12-reference template for G component of the concatenated SCID images. In this template, reference pixels are selected from both B and G components. Table 4 shows the entropies of the G and R components of the image "cafeteria" conditioned by the multi-component 12-reference template (template M), compared with those conditioned by the component specific 12-reference template (template S). From this table, it is observed that the entropy can be reduced by about 8% with the multi-component N-reference template.

D00	D01	D02	D03	D04
D10	D11	D12	D13	D14
D20	D21	D22	D21	D22

Figure 6. New base window for the component previously transmitted

plane B

7	5	2	-	-

plane G

			11	10	4	8
3	-	-	12	-	-	-
1	9	6	Xj			

Figure 7. Priority order of the top 12 reference pixels in the two base windows obtained by the EPM

Table 4. Conditional entropies with two types of 12-reference template S and M:
Template S refers only to pixels in the component currently encoded, Template M can refer to pixels in the component previously transmitted.

cafeteria	S	M	M/s
G (ref. B)	0.798	0.731	0.916
R (ref. G)	0.752	0.694	0.923

Encoding sequence of components

In transmitting these three color components of an image, there are six possible sequences; they are (B, G, R), (B, R, G), (G, B, R), (G, R, B), (R, B, G) and (R, G, B). Though the entropy would not be affected so much by the sequences, theoretically, it would be needed to know the actual tendency under the limited hard ware concerning the sequences. In this section, we discuss the best encoding sequence of components as well as the best combination of the N-reference templates for minimizing the total transmission time of three color components.

As the first step to solve this problem, multi-component N-reference templates are examined; e.g., for encoding G component, a template which includes reference pixels from G and B components and one which includes G and R components are examined. Though it is possible to refer to the other two components for the third component, the calculation of the entropy is not took place for its complexity. Then, the entropy of the first component conditioned by its component specific N-reference template and those of the second and third components conditioned by multi-component N-reference templates are examined to find the best method for each of six transmission sequences.

Figure 8 shows the result, in case of the concatenated image of three SCID images. In Figure 8, "XrY" denotes that the multi-component N-reference template for X component includes the reference pixels in both X and Y components, and the three symbols in the parentheses denote the transmission sequence; e.g., (B, GrB, RrG) means that the B component is encoded first, G component is encoded next with the template which includes reference pixels from G and B components, and finally the R component is encoded with referring to the G and R components. From this figure, it is observed that the average entropy of three color components for each sequence is almost equal to each other, and that (B, GrB, RrG) and (R, GrR, BrG) show a slightly better performance.

Since G component is generally more close to luminance component than the other two components; it generally carries more information than the others. Suppose that G component is encoded first, the entropy of the first component is larger than the case when B or R component is encoded first, while the entropies of B and R components can be reduced with referring to G component. On the other hand, in case of transmitting B or R component first, the entropy of the first component is not so large, while the entropy of G component is still large even referring to B or R component.

As a common transmission sequence and a common set of N-reference templates, we choose (B, GrB, RrG) because it shows slightly better performance than the others. Table 5 shows the average entropies of three color components conditioned by the 10-reference templates optimized for the common transmission sequence; the overall benefit applying multi-component is around 3%.

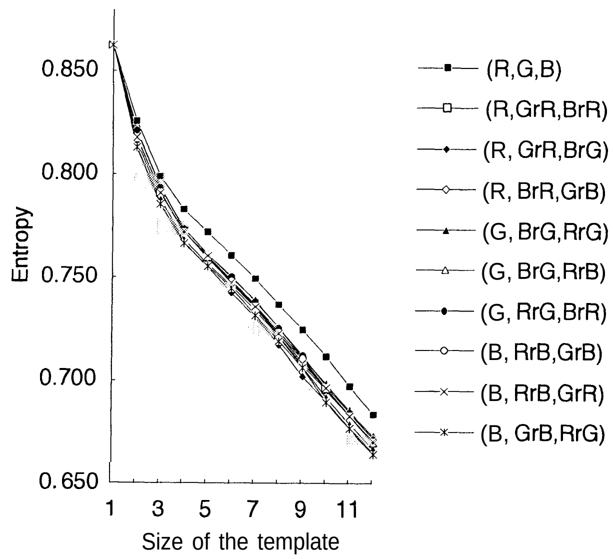

Figure 8. Average entropy of 3 component for each transmission sequence

Table 5. Average conditional entropies of three color components with two types of 10-reference templates:
S ; component specific 10-reference template.
M ; multi component 10-reference template.

	S	M	M/S
bicycle	0.645	0.629	0.975
cafeteria	0.779	0.735	0.944
portrait	0.665	0.658	0.989
average	0.696	0.674	0.969

Summary and conclusions

In this paper, a method for searching a quasi-optimal template of reference pixels in encoding hi-level images quantized by the MAEM is evaluated. It is confirmed that the method provides a template which produces almost minimum entropy, with much less calculation load, and that the method provides a template commonly applicable to a wide variety of monochrome images.

Since three components of a half-toned color image by the MAEM are correlated with each other, the above method is extended to obtain a template of reference pixels which consists of pixels in the component currently encoded as well as those in the component previously encoded. Simulation results show that the advantage of utilizing correlation between different color components in encoding them is marginal. Finally, it can be said that the straight forward extension of the method encoding monochrome half-tone to color half-tone images provides good result, however, to utilize the correlation between the components does not provide a significant improvement. This item should be a future research item.

References

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